STAT 140: Probability
Extra Problem Solution

Leishmaniasis is a neglected tropical disease that places a billion people at risk each year. The most severe form of the disease is visceral, meaning that it attacks internal organs and can cause death. In Brazil, the probability of a person in the general population having disease is 10 percent. A diagnostic test is used to test for disease. If a person has the disease, the probability is 91 percent that they will have a positive diagnostic test result. If a person does not have disease, the probability is 8 percent that they will have a positive diagnostic test result.

- Let D be the event that a person has leishmaniasis.
- Let D^c be the event that a person does not have leishmaniasis.
- Let T be the event that they test positive on a diagnostic test.
- Let T^c be the event that they do not test positive on a diagnostic test (the complement of T).

Find the following probabilities. Use appropriate notation where needed.

(a) $P(D)$

$$P(D) = 0.10$$

(This is the prevalence of the disease; it is a marginal probability.)

(b) $P(T|D)$

$$P(T|D) = 0.91$$

(This is the sensitivity of the diagnostic test; it is a conditional probability.)

(c) $P(T|D^c)$

$$P(T|D^c) = 0.08$$

(This is 1-specificity of the diagnostic test; it is a conditional probability.)

(d) $P(T \text{ and } D)$

$$\begin{aligned} P(T \text{ and } D) &= P(T|D) \times P(D) \\ &= 0.91 \times 0.10 \\ &= 0.091 \end{aligned}$$

(This is a joint probability; we use the general multiplication rule to find this probability.)

(e) $P(T \text{ and } D^c)$

$$\begin{aligned} P(T \text{ and } D^c) &= P(T|D^c) \times P(D^c) \\ &= P(T|D^c) \times (1 - P(D)) \\ &= 0.08 \times 0.90 \\ &= 0.072 \end{aligned}$$

(This is a joint probability; we use the general multiplication rule to find this probability. We also have to use the complement rule to find $P(D^c)$ using the prevalence.)

(f) $P(T)$

$$\begin{aligned} P(T) &= P(T \text{ and } D) + P(T \text{ and } D^c) \\ &= 0.091 + 0.072 = 0.163 \end{aligned}$$

(This is a marginal probability; we use the Law of Total Probability to find this marginal probability.)

(g) $P(D|T)$

$$\begin{aligned} P(D|T) &= \frac{P(D \text{ and } T)}{P(T)} \\ &= \frac{0.091}{0.163} \\ &= 0.558 \end{aligned}$$

(This is a conditional probability, which we calculate using Bayes' Rule.)

What are the chances that a person who tests positive actually has leishmaniasis?

The probability that a person who tests positive actually has leishmaniasis is 55.8%.